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Ferrimagnetism in a disordered binary alloy

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Abstract. The magnetic properties (transition temperature, compensation temperature and magnetizations) of a disordered ferrimagnetic binary alloy are studied by the use of two theoretical frameworks, namely the standard mean-field theory and the effective-field theory with correlations. We find a number of outstanding phenomena in ferrimagnetism, such as the *possibility of many compensation points in the total magnetization curve* and magnetization curves not predicted in the Néel theory.

1. Introduction

Ferrimagnetism has been intensively investigated in the past, both experimentally and theoretically. Ferrimagnets have several sublattices with a finite resultant moment, such as spinel ferrites, garnet ferrites and many other substances. However, they normally take complicated magnetic structures [1]. Because of the complexity of the structures, very little work has been done in trying to apply more advanced theories to ferrimagnetism beyond the standard mean-field theory approximation (MFA) [2].

On the other hand, because of the difficulties inherent in the theoretical description of such complicated ferrimagnetic systems, it is sometimes necessary to make some simplifications. Theoretically, a binary ferrimagnetic alloy model of the type A_pB_{1-p} randomly occupied by the two different species (A and B) of magnetic ions has been extensively investigated, in order to analyse the magnetic properties of ferrimagnetic alloys. Here, A and B ions have different spin values S_A and S_B ($S_A \neq S_B$). Even for the simplified model, the MFA has often been used for analyses of the experimental data. However, it is known that the MFA argument has some deficiencies, owing to the neglect of spin correlations, such as the disappearance of a disordered (or paramagnetic) phase [3].

A further but not essential simplification is to assume that the interactions between spins in the disordered binary alloy are of the Ising form with the exchange interactions different for different pairs of spins. Let the A and B atoms have $S_A = 1/2$ and $S_B > S_A$. The Hamiltonian of the system is given by

$$\mathcal{H} = -\sum_{i<j}[J_A\delta_{iA}\delta_{jA} + J_B\delta_{iB}\delta_{jB} + J_{AB}(\delta_{iA}\delta_{jB} + \delta_{iB}\delta_{jA})]S_i^zS_j^z\xi_i\xi_j - D\sum_i(S_i^z)^2\delta_{iB}\xi_i. \quad (1)$$

Here the first sum is over all nearest-neighbour pairs; the spin of A atoms can be $+1/2$ or $-1/2$ and that of B atoms takes the $(2S_B+1)$ values allowed for a spin S_B; D is the single-ion anisotropy of B atoms and $\delta_{i\alpha}$ (α = A or B) expresses that a site i is occupied by an atom of type α; the $J_{\alpha\beta}$ ($J_{\alpha\alpha} = J_\alpha$) are the exchange interactions between type-α and type-β atoms; and ξ_i is a random variable, which takes the value of unity or zero. In the previous work [4], one of the present authors has examined the magnetic properties of a ferrimagnetic binary alloy (A_pB_{1-p}) with $S_A = 1/2$ and $S_B = 1$ described by (1), using the effective-field theory with correlations (EFT) [5], superior to the standard MFA.

Very recently, we have discussed a new formulation for treating the pure ($p = 0.0$) Ising system with a higher spin value, $S_B = 1, 3/2, 2, \ldots$ (or the spin-S Blume–Capel model [6]). In the work, we have found that the physical properties of the pure system are completely different in the negative region of D, depending on whether the spin value S_B takes an integer spin or a half-integer spin. For an integer spin, the tricritical behaviour may appear when D takes a negative value. For a large negative value of D, furthermore, the system is in the $S_i^z = 0$ state, so that it does not show any long-range order. On the other hand, for a half-integer spin ($S_B > 1/2$) the magnetic properties may be changed continuously with the value of D except the magnetization at $T = 0$ K. Thus, the results indicate that the magnetic properties of a binary ferrimagnetic alloy may take different behaviours depending on the value of S_B.

In this work, we study via the new theory [7] the magnetic properties (phase diagram and magnetization) of a ferrimagnetic binary alloy described by (1) when the value of S_B is selected as $S_B = 3/2$. A number of outstanding phenomena are obtained. The outline of this work is as follows. In section 2, the two formulations of the problem are given briefly on the basis of the standard MFA and the new formulation (EFT), superior to the MFA. In section 3, the phase diagram is examined. In sections 4 and 5, the possibility of many compensation points is investigated by the use of the two formulations. Magnetization curves not predicted in the Néel theory of ferrimagnetism are also found.

2. Formulation

We consider a binary two-sublattice ferrimagnetic Ising system (or A_pB_{1-p}) with $J_{AB} < 0$. Let the A and B atoms have different spins ($S_A = 1/2$ and $S_B = 3/2$). The Hamiltonian of the system is given by (1). Here, performing the random average (or $\langle\ldots\rangle_r$), the averaged value of ξ_i has a restriction

$$\langle\xi_i\delta_{iA}\rangle_r + \langle\xi_i\delta_{iB}\rangle_r = 1 \tag{2}$$

where $\langle\xi_i\delta_{iA}\rangle_r = p$ is the concentration of A atoms.

Let us first start on the standard MFA argument, in order to clarify how the magnetizations can be formulated. In the MFA, the averaged magnetizations of A and B atoms are given by

$$m_A = \langle\langle S_i^z\delta_{iA}\xi_i\rangle\rangle_r / \langle\delta_{iA}\xi_i\rangle_r \tag{3a}$$

$$= \tfrac{1}{2}\tanh(\tfrac{1}{2}\beta E_A) \tag{3b}$$

and

$$m_B = \langle\langle S_i^z\delta_{iB}\xi_i\rangle\rangle_r / \langle\delta_{iB}\xi_i\rangle_r \tag{4a}$$

$$= f(E_B) \tag{4b}$$

with

$$E_A = pzJ_Am_A + (1-p)zJ_{AB}m_B \qquad E_B = pzJ_{AB}m_A + (1-p)zJ_Bm_B \tag{5}$$

where $\beta = 1/k_BT$, z is the coordination number and the function $f(x)$ is defined by

$$f(x) = \frac{1}{2}\frac{3\sinh(3\beta x/2) + \exp(-2\beta D)\sinh(\beta x/2)}{\cosh(3\beta x/2) + \exp(-2\beta D)\cosh(\beta x/2)}. \tag{6}$$

Then, the total magnetization M per site is given by

$$M = pm_A + (1-p)m_B. \tag{7}$$

On the other hand, by the use of the formulations in [4, 7] the sublattice magnetizations m_A and m_B defined by (3a) and (4a) are given by

$$m_A = \{p[\cosh(J_A\nabla/2) + 2m_A \sinh(J_A\nabla/2)] + (1-p)[\cosh(J_{AB}\eta\nabla) + (m_B/\eta)\sinh(J_{AB}\eta\nabla)]\}^z F_A(x)|_{x=0} \tag{8}$$

and

$$m_B = \{p[\cosh(J_{AB}\nabla/2) + 2m_A \sinh(J_{AB}\nabla/2)] + (1-p)[\cosh(J_B\eta\nabla) + (m_B/\eta)\sinh(J_B\eta\nabla)]\}^z F_B(x)|_{x=0} \tag{9}$$

where $\nabla = \partial/\partial x$ is a differential operator and the functions $F_A(x)$ and $F_B(x)$ are respectively defined by $F_A(x) = \frac{1}{2}\tanh(\beta x/2)$ and $F_B(x) = f(x)$. As discussed in [7], the parameter η in (8) and (9) is given by

$$\eta^2 = \langle\langle (S_i^z)^2 \xi_i \delta_{iB}\rangle\rangle_r / \langle \xi_i \delta_{iB}\rangle_r = \{p[\cosh(J_{AB}\nabla/2) + 2m_A \sinh(J_{AB}\nabla/2)] + (1-p)[\cosh(J_B\eta\nabla) + (m_B/\eta)\sinh(J_B\eta\nabla)]\}^z G_B(x)|_{x=0} \tag{10}$$

where the function $G_B(x)$ is defined by

$$G_B(x) = \frac{1}{4}\frac{9\cosh(3\beta x/2) + \exp(-2D\beta)\cosh(\beta x/2)}{\cosh(3\beta x/2) + \exp(-2D\beta)\cosh(\beta x/2)}. \tag{11}$$

In order to derive these equations, the exact Ising spin identities were applied to the evaluation of the sublattice magnetizations (3a) and (4a) for $S_A = 1/2$ and $S_B = 3/2$. The decoupling approximation was introduced for treating the multispin correlation functions, namely

$$\langle\langle \xi_i S_i^z \xi_j S_j^z \ldots \xi_k S_k^z \rangle\rangle_r \simeq \langle\langle \xi_i S_i^z\rangle\rangle_r \langle\langle \xi_j S_j^z\rangle\rangle_r \ldots \langle\langle \xi_k S_k^z\rangle\rangle_r \tag{12}$$

for $i \neq j \neq \ldots \neq k$. As discussed in recent work [7, 8], the statistical accuracy (of (12)) corresponds to the Zernike approximation [9] of a spin-1/2 Ising model.

We are now interested in studying the transition temperature T_C and the compensation temperature T_{comp} of the disordered ferrimagnetic alloy. In order to determine the transition temperature, the usual argument that the sublattice magnetizations tend to zero as the temperature approaches a critical temperature allows us to consider only terms linear in the sublattice magnetizations m_A and m_B. In the mean-field theory (or (3b) and (4b)), the critical surface characterizing the ferrimagnetic phase stability limit is determined by

$$(\tfrac{1}{4}pt - 1) - [\tfrac{1}{4}t(1-p)a\gamma - 1] = (\tfrac{1}{4}t)^2 p(1-p)a\delta^2 \tag{13}$$

with

$$t = zJ_A/k_BT \qquad \delta = J_{AB}/J_A \qquad \gamma = J_B/J_A \qquad d = D/zJ_A \tag{14}$$

where the parameter a in (13) is defined as

$$a = [9 + \exp(-2td)]/[1 + \exp(-2td)]. \tag{15}$$

Within the new formulation (EFT), the phase diagram (or T_C) can be determined from

$$(2pK_1 - 1)[(1-p)L_2 - 1] = 2p(1-p)L_1K_2 \tag{16}$$

with

$$
\begin{aligned}
K_1 &= z \sinh(J_A \nabla/2)[p \cosh(J_A \nabla/2) + (1-p)\cosh(J_{AB}\eta_0 \nabla)]^{z-1} F_A(x)|_{x=0} \\
K_2 &= (z/\eta_0) \sinh(J_{AB}\eta_0 \nabla)[p \cosh(J_A \nabla/2) + (1-p)\cosh(J_{AB}\eta_0 \nabla)]^{z-1} F_A(x)|_{x=0} \\
L_1 &= z \sinh(J_{AB} \nabla/2)[p \cosh(J_{AB} \nabla/2) + (1-p)\cosh(J_B\eta_0 \nabla)]^{z-1} F_B(x)|_{x=0} \\
L_2 &= (z/\eta_0) \sinh(J_B\eta_0 \nabla)[p \cosh(J_{AB} \nabla/2) + (1-p)\cosh(J_B\eta_0 \nabla)]^{z-1} F_B(x)|_{x=0}
\end{aligned}
\tag{17}
$$

where η_0 *is the solution of*

$$\eta_0^2 = [p \cosh(J_{AB} \nabla/2) + (1-p)\cosh(J_B\eta_0 \nabla)]^z G_B(x)\big|_{x=0}. \tag{18}$$

The coefficients K_1, K_2, L_1 and L_2 can be easily calculated by the use of a mathematical relation $\exp(a\nabla)\Phi(x) = \Phi(x+a)$.

In a ferrimagnet, there is an interesting possibility of the existence, under certain conditions, of a compensation temperature at which the resultant magnetization vanishes. The appearance of a compensation point is due to the fact that the magnetic moments of the sublattices compensate each other completely at $T = T_{comp}$, owing to the different temperature dependences of the sublattice magnetizations. Thus, the compensation temperature can be determined by introducing the condition $M = 0$ into the coupled equations (8) and (9) (or (3*b*) and (4*b*)).

We are now able to study the magnetic properties by the use of the equations derived above. Before discussing them, let us here examine some mathematical structure of the present formulation. When $D \to -\infty$, the functions $G_B(x)$ and $F_B(x)$ are given by

$$G_B(x) = \tfrac{1}{4} \qquad \text{and} \qquad F_B(x) = \tfrac{1}{2}\tanh(\beta x/2) \tag{19}$$

from which the parameter η_0 is

$$\eta_0 = \tfrac{1}{2} \qquad \text{for } D/J_A \to -\infty. \tag{20}$$

Thus, it indicates that only the $S_i^z = \pm 1/2$ state is allowed on B atoms. Then, the present problem reduces to that of a mixed spin-1/2 Ising alloy. In particular, the relation (16) becomes equivalent to that of the mixed spin-1/2 alloy (equation (8) in [10]) except that each spin in [10] is defined by $S_i^z = \pm 1$. In contrast to the previous work [4] for the case of $S_B = 1$, one can compare the results of the present problem with some known ones of the mixed spin-1/2 Ising alloy, when necessary, by taking the limit.

In this section, the general frameworks based on the two formulations have been presented, from which the phase diagram (T_C and T_{comp}) and the total and sublattice magnetizations in a ferrimagnetic binary alloy with a coordination number z can be evaluated. In the following sections, we shall study the physical quantities of the ferrimagnet with $z = 3$ (or for the honeycomb lattice) by solving them numerically.

3. Phase diagram

In this section, let us examine the phase diagrams of the ferrimagnetic binary alloy with $z = 3$. However, we have four parameters (J_A, J_B, J_{AB}, D) for the numerical evaluations. In order to relate the present results with those in real ferrimagnetic alloys, let us take $J_A > J_B$ in the following. In real amorphous rare earth (RE)–transition metal (TM) ferrimagnetic alloys [3], J_A, J_{AB} and J_B correspond to TM–TM, RE–TM and RE–RE interactions, respectively. Then, the magnitudes of exchange interactions are usually taken as $0 < J_B < -J_{AB} < J_A$.

In figure 1, we first show a typical phase diagram obtained from the MFA (or (13) and $M = 0$) for the system with $J_B/J_A = 0.1$ and $D/J_A = -2.0$. In the following, broken curves denote the change of T_C and full curves represent the variations of T_{comp}. In the figure, T_C or T_{comp} versus p are plotted by selecting four values of J_{AB}/J_A. Here, notice that for $J_{AB} = 0.0$ the T_C curve takes a finite value at each value of p and the T_{comp} cannot be obtained.

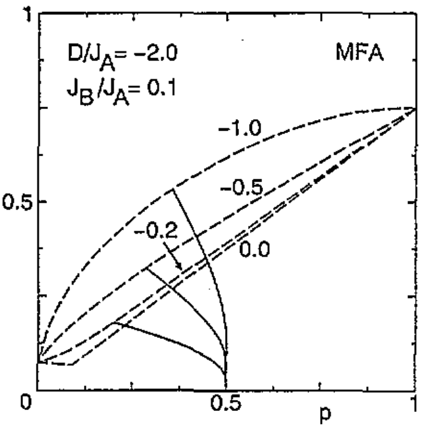
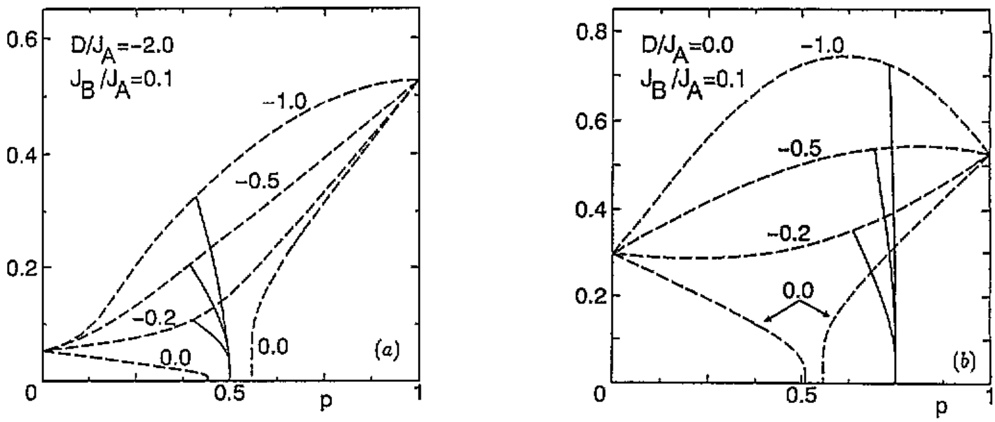


Figure 1. The phase diagram of the disordered ferrimagnetic binary alloy with $z = 3$ obtained from the standard mean-field theory (MFA), equations (3b) and (4b), when the parameters, J_B/J_A and D/J_A, are selected as $J_B/J_A = 0.1$ and $D/J_A = -2.0$ and the value of J_{AB}/J_A is $J_{AB}/J_A = -1.0, -0.5, -0.2$ or 0.0. The full and broken curves represent respectively the compensation temperature T_{comp} and Curie temperature T_C of the system.

In figure 2, on the other hand, the phase diagrams obtained from the EFT (or (16) and $M = 0$) are plotted for the system with $J_B/J_A = 0.1$ by taking the three values of D/J_A and selecting the same values of J_{AB}/J_A as in figure 1. In figure 2(a), the same T_C, T_{comp} versus p plots as those of figure 1 are given by selecting the value of $D/J_A = -2.0$. Comparing the results of figure 2(a) with those of figure 1, the whole behaviours are qualitatively similar except for the results labelled $J_{AB}/J_A = 0.0$. In figure 2(a) the dashed curve (or T_C) with $J_{AB}/J_B = 0.0$ reduces to zero in the region of p near $p = 0.5$, which is in sharp contrast to the corresponding curve of figure 1. However, the result of figure 2(a) is reasonable, since for $J_{AB} = 0.0$ the sublattice magnetization m_A is decoupled from that of m_B and hence the system becomes similar to the usual dilution problem. Thus, by going from the MFA to the present formulation (EFT) one can refine the fault of the MFA, since the EFT includes automatically some correlations through the usage of the Van der Waerden identities in [4, 7].

In figures 2(b) and (c), the corresponding phase diagrams of the EFT are shown by taking the values of $D/J_A = 0.0$ and $D/J_A = 2.0$. Here, one should notice the characteristic behaviours of T_{comp} (full curves). In figure 2(a), the T_{comp} for the system with $D/J_A = -2.0$ can be obtained in the region of $p \leqslant 0.5$ and reduces to zero at $p = 0.5$. On the other hand, for the system with $D/J_A \geqslant 0.0$ it can be obtained in some region of $p > 0.5$. Moreover, the concentration dependence of T_{comp} for figure 2(c) is clearly different from that for figure 2(a). Also, the T_{comp} curves in figures 2(b) and (c) reduce to zero at $p = 0.75$. Thus, depending on the sign of D, the compensation point in the binary ferrimagnetic alloy can be obtained in a different region of p. In particular, the T_{comp} curve reduces to zero at $p = 0.5$ for $D/J_A = -2.0$ or $p = 0.75$ for $D/J_A \geqslant 0.0$. The phenomenon can be explained as follows. For $D/J_A = -2.0$, the spin on the B atoms is in the $S_i^z = \pm 1/2$ state, so that at $T = 0$ K the compensation point can be found at $p = 0.5$ because of the relation $\frac{1}{2}p - \frac{1}{2}(1-p) = 0$. For $D/J_A \geqslant 0.0$, on the other hand, the spin on the B atoms is in the $S_i = \pm 3/2$ state and hence the compensation point at $T = 0$ K is obtained at $p = 0.75$ due to $\frac{1}{2}p - \frac{3}{2}(1-p) = 0$.



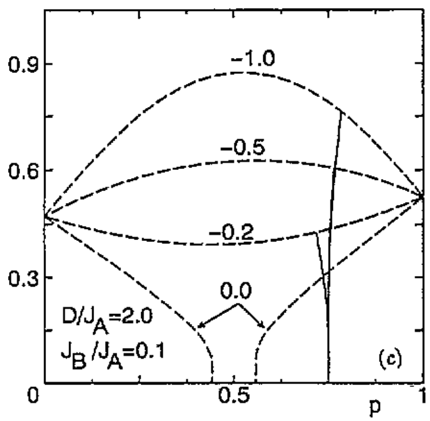


Figure 2. The phase diagrams of the disordered ferrimagnetic binary alloy with $z = 3$ and $J_B/J_A = 0.1$ obtained from the effective-field theory (EFT), equations (8)–(10), when the value of J_{AB}/J_A is $J_{AB}/J_A = -0.1, -0.5, -0.2$ or 0.0. T_{comp} and T_C are represented as full and broken curves, respectively. (*a*) The phase diagram obtained for $D/J_A = -2.0$, which should be compared with figure 1 for the MFA. (*b*) The phase diagram for $D/J_A = 0.0$. (*c*) The phase diagram for $D/J_A = 2.0$.

In this section, the T_C or T_{comp} versus p curves in the binary ferrimagnetic alloy have been examined by the use of the two formulations, namely the MFA and the EFT. All of these results indicate that the magnetic moments of the sublattices compensate each other completely at $T = T_{comp}$, above which temperature compensation is no longer obtained and the resultant moment does not disappear until the Curie point T_C is reached. That is to say, these results mean that only one compensation point in a binary ferrimagnetic alloy may exist in the region of $T \leqslant T_C$, as usually written in standard textbooks of magnetism (such as [1, 3]). In the following sections, however, we shall discuss whether the concept of only one compensation point has to be changed.

4. The possibility of many compensation points

Recently, ferrimagnetic RE–TM based multilayer systems have been of considerable interest because of their potential device applications. The layer-thickness dependence of T_{comp} is clarified in some experimental data [11]. For the heavy RE, the 3d–4f interlayer indirect interaction is negative and hence some of them possess a compensation-point temperature when the thickness is not too thick. In relation to these experimental works, we have very recently studied the magnetic properties of some ferrimagnetic multilayer systems with different spin values by the use of the new formulation [7]. In the process, we have found the existence of two compensation points in multilayer systems consisting of spin-1/2 and spin-3/2 components, when the single-ion anisotropy D is selected as a negative value [12].

In the following, let us clarify whether more than one compensation temperature may be obtained in a binary ferrimagnetic alloy by the use of the same frameworks as in the previous sections. For this, let us first start on the MFA argument and direct our attention to the concentration near $p = 0.75$. A typical phase diagram for the system with $D/J_A = 0.0$ and $J_B/J_A = 0.5$ is shown in figure 3, selecting three negative values of J_{AB}/J_A. The results clearly show that two compensation points may exist in the system with a certain concentration very near to $p = 0.75$. In figure 4, therefore, the phase diagram as well as the magnetization curves for the system with $z = 3$ are examined by taking $p = 0.745$. In figure 4(a), the phase diagram in the $(T, |J_{AB}|/J_A)$ space is plotted by selecting three values of D/J_A, where the full and broken curves also represent T_{comp} and T_C, respectively. As is seen from the figure, T_{comp} curves indicate that two compensation points are possible in the system with a special value of $|J_{AB}|/J_A$, such as $0.203 < |J_{AB}|/J_A < 0.343$ for $D/J_A = 0.0$.

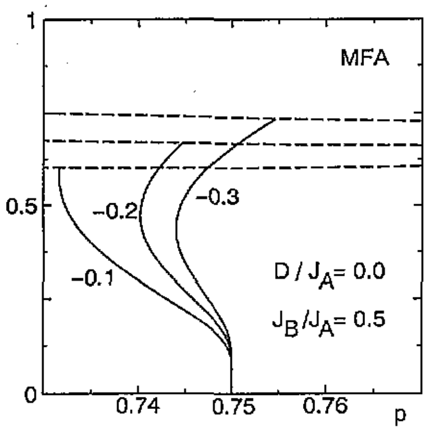
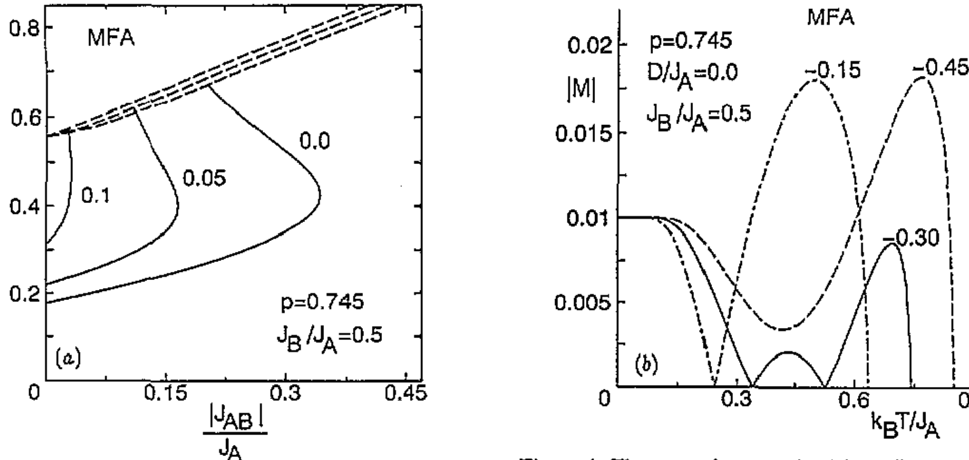


Figure 3. The phase diagram of the disordered ferrimagnetic binary alloy with $z = 3$ and concentration near $p = 0.75$ obtained from the MFA, when the parameters J_B/J_A and D/J_A are fixed at $J_B/J_A = 0.5$ and $D/J_A = 0.0$, and the value of J_{AB}/J_A is $J_{AB}/J_A = -0.1$, -0.2 or -0.3.

On the other hand, in order to confirm the prediction for the existence of two compensation points, the temperatue dependences of $|M|$ in the system with $D/J_A = 0.0$ are shown in figure 4(b) by selecting three values of J_{AB}/J_A, namely $J_{AB}/J_A = -0.15$, -0.3 and -0.45. For $J_{AB}/J_A = -0.15$, only one compensation point is obtained. For $J_{AB}/J_B = -0.30$, as predicted in figure 4(a), two compensation points are found. For $J_{AB}/J_A = -0.45$, the magnetization curve shows a minimum and a maximum in the temperature region below T_C. As depicted in figure 4(c), however, the sublattice magnetizations m_A and m_B show the normal behaviour in their thermal variations.

Here, one has to notice that the above results in figure 4(a) based on the MFA argument are not correct for $J_{AB} = 0.0$. Even for $J_{AB} = 0.0$, the sublattice magnetization m_B may take a finite value and hence T_{comp} takes a different value depending on the value of D. For $p = 0.745$, the concentration of B atoms should be lower than the critical concentration p_c (roughly, $p_c \simeq 2/z = 0.667$). For $J_{AB} = 0.0$, therefore, m_B must be zero.

In order to refine the fault of the MFA argument, we have also investigated the same system as that of figure 4(a) by the use of the new formulation (EFT). The results are depicted in figure 5. In figure 5(a), the phase diagram is shown. In contrast to figure 4(a), all T_{comp} curves in figure 5(a) go to zero when J_{AB} goes to zero. In particular, when $J_{AB} = 0.0$, the sublattice magnetization m_B is given by $m_B = 0.0$. The open circle at $J_{AB} = 0.0$ denotes the fact. That is to say, the open circle is a singular point for T_{comp}. In figure 5(b), on the other hand, the temperature dependences of $|M|$ for the system with $D/J_A = 0.0$ are depicted



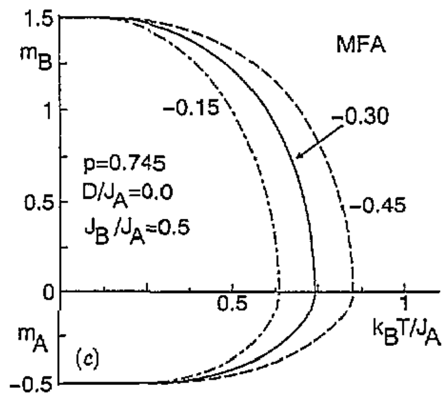


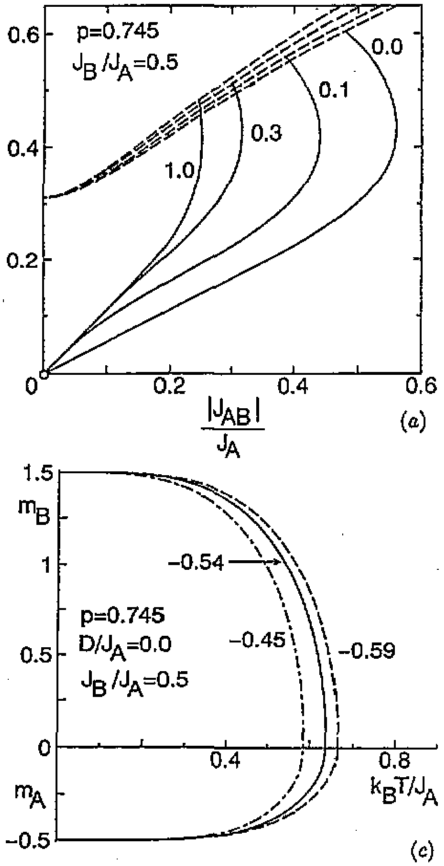
Figure 4. The magnetic properties (phase diagram and magnetization curves) of the disordered ferrimagnetic binary alloy with $z = 3$ obtained by the use of the MFA, when the values of p and J_B/J_A are fixed at $p = 0.745$ and $J_B/J_A = 0.5$. (*a*) T_{comp} (full curves) and T_C (broken curves) of the system for three values of D/J_A ($D/J_A = 0.0, 0.05, 0.1$) depicted as a function of $|J_{AB}|/J_A$. (*b*) The $|M|$ versus T curves in the alloy with $D/J_A = 0.0$, when three values of J_{AB}/J_A are selected: $J_{AB}/J_A = -0.15$ (chain curve), $J_{AB}/J_A = -0.30$ (full curve) and $J_{AB}/J_A = -0.45$ (broken curve). (*c*) The thermal variations of sublattice magnetizations in the system with $z = 3$ for three values of J_{AB}/J_A : $J_{AB}/J_A = -0.15$ (chain curve), $J_{AB}/J_A = -0.30$ (full curve) and $J_{AB}/J_A = -0.45$ (broken curve).

by selecting the three values of $|J_{AB}|/J_A$, namely $J_{AB}/J_A = -0.45$, -0.54 and -0.59. The results of figure 5(*b*) are very similar to those of figure 4(*b*). For $J_{AB}/J_A = -0.54$, we can find two compensation points in the magnetization curve. Figure 5(*c*) shows the temperature dependences of the sublattice magnetizations, which also express the normal behaviour.

In this section, we have studied the possibility of two compensation points in the binary ferrimagnetic alloy with $z = 3$. As shown in figures 3–5, the binary ferrimagnetic alloy with an appropriate condition near $p = 0.75$ may show two compensation points in the total magnetization curve. The phenomenon has not been predicted in the Néel theory of ferrimagnetism. Furthermore, the magnetization curve exhibiting a minimum and a maximum, such as the curve labelled -0.45 in figure 4(*b*) or the curve labelled -0.59 in figure 5(*b*), has not been predicted in the Néel theory.

5. Other new phenomena

In section 4, we have examined in detail the phase diagram as well as the magnetization curves in the system with a concentration very near $p = 0.75$, from which some new phenomena in ferrimagnetism have been found. These results indicate that one should also look at the system with a concentration very near $p = 0.5$. As discussed in section 3,



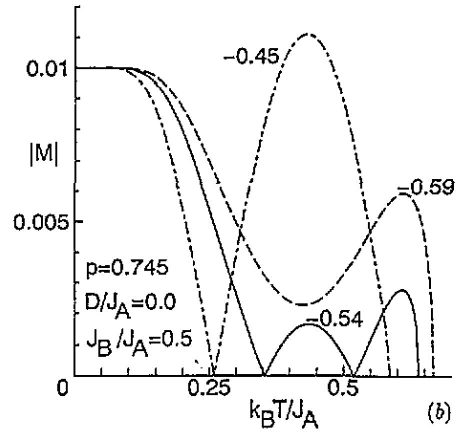


Figure 5. The magnetic properties of the same system as that of figure 4 investigated by the use of the effective-field theory (EFT). (*a*) T_{comp} (full curve) and T_C (broken curve) of the system with $z = 3$ plotted as a function of $|J_{AB}|/J_A$, when the value of D/J_A is $D/J_A = 0.0$, 0.1, 0.3 or 1.0. (*b*) The $|M|$ versus T curves in the system with $D/J_A = 0.0$, when three values of J_{AB}/J_A are selected: $J_{AB}/J_A = -0.45$ (chain curve), $J_{AB}/J_A = -0.54$ (full curve) and $J_{AB}/J_A = -0.59$ (broken curve). (*c*) The thermal variations of sublattice magnetizations in the same systems as those of (*b*).

however, the MFA argument is not appropriate for such investigation, since for $J_{AB} = 0.0$ it predicts an incorrect result for T_C. In this section, let us examine the magnetic properties of the system with $z = 3$ by the use of the EFT, especially for negative values of D/J_A.

Figure 6 shows the two typical phase diagrams of the binary ferrimagnetic alloy with fixed values $(J_B/J_A, D/J_A)$, namely (0.1, −2.0) for figure 6(*a*) and (0.5, −0.3) for figure 6(*b*), in the (T, p) space very near to $p = 0.5$, when the value of J_{AB}/J_A is changed. In each figure, broken and full curves also represent T_C and T_{comp}, respectively. In particular, notice that these phase diagrams indicate the possibility for the existence of three compensation points as well as one compensation point and two compensation points in the system, when a special value of p is selected in the regions.

In order to confirm the prediction of three or two compensation points, the total magnetization curves of the system with $p = 0.495$ are plotted in figure 7 by selecting the special set of values $(J_B/J_A, D/J_A, J_{AB}/J_A)$ in figure 6, namely (0.1, −2.0, −2.465) for figure 7(*a*), (0.5, −3.0, −3.08) for the full curve in figure 7(*b*) and (0.1, −2.0, −2.49) for the broken curve in figure 7(*b*). As is seen from the figures, three or two compensation points are obtained. On the other hand, when other sets of values are selected from figure 6, one can find some characteristic magnetization curves, like the chain curves in figures 4(*b*) and 5(*b*). The results are drawn in figure 8. In figure 8(*a*), the temperature dependences of $|M|$ for the system with $p = 0.495$ are shown, when the two sets of $(J_B/J_A, D/J_A, J_{AB}/J_A)$ are selected. The results may exhibit a compensation point, but above this temperature the

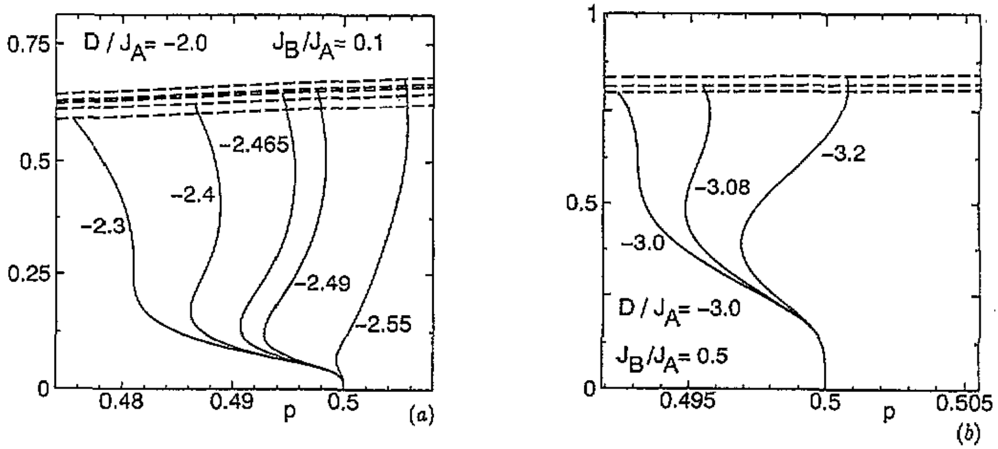


Figure 6. The phase diagrams of the disordered ferrimagnetic binary alloy with $z = 3$ and the concentration near $p = 0.5$ obtained from the EFT. (a) The phase diagram for the system with $J_B/J_A = 0.1$ and $D/J_A = -2.0$, when the value of J_{AB}/J_A is $J_{AB}/J_A = -2.3, -2.4, -2.465, -2.49$ and -2.55. (b) The phase diagram for the system with $J_B/J_A = 0.5$ and $D/J_A = -3.0$, when the value of J_{AB}/J_A is $J_{AB}/J_A = -3.0, -3.08$ and -3.2.

magnetization curve may express a weak minimum and a weak maximum until the Curie point is reached. In figure 8(b), the magnetization curves of the systems with $p = 0.495$ and other sets of values express a minimum and a maximum, like the broken curves in figures 4(b) and 5(b).

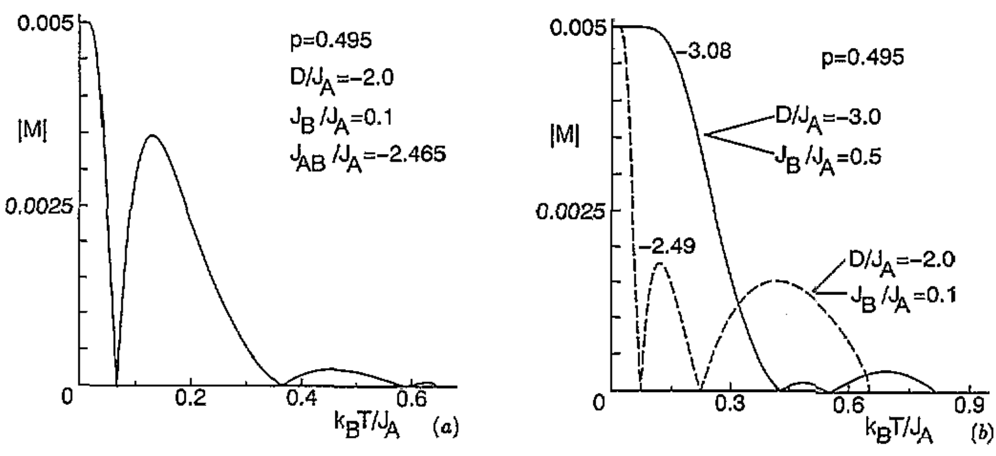


Figure 7. The temperature dependences of $|M|$ in the ferrimagnetic alloy with $z = 3$ and $p = 0.495$ obtained from the EFT. (a) The $|M|$ curve of the system with the set of parameters $(J_B/J_A, D/J_A, J_{AB}/J_A) = (0.1, -2.0, -2.465)$. ($b$) The $|M|$ curves of two systems with the sets of parameters $(J_B/J_A, D/J_A, J_{AB}/J_A) = (0.5, -3.0, -3.08)$ for the full curve and (0.1, -2.0, -2.49) for the broken curve.

As noted in section 3, when the value of D is $D/J_A \leqslant -2.0$ and $p = 1/2$, the ground state of the binary ferrimagnetic alloy must exactly be in the $S_i^z = \pm 1/2$ state both for A and B atoms and hence the total magnetization M takes the value $M = 0$ at $T = 0$ K. Accordingly, the thermal variations of $|M|$ in the systems with $p = 0.5$ may

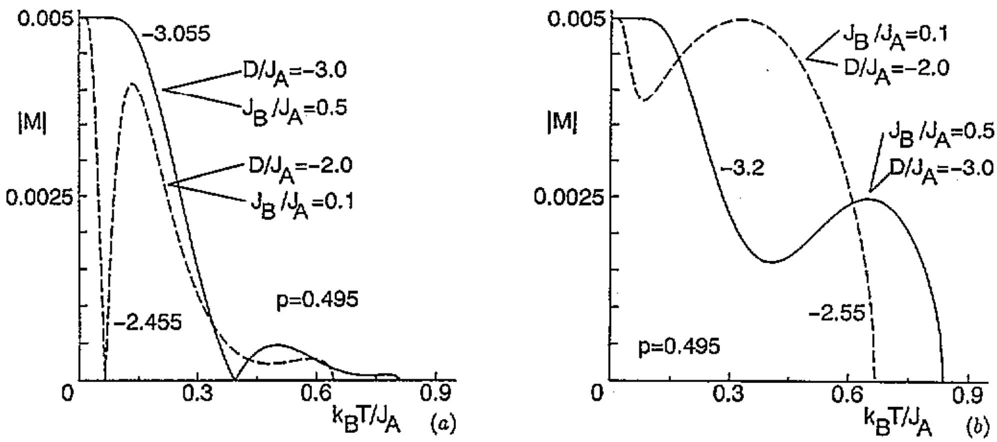


Figure 8. Some characteristic temperature dependences of $|M|$ in the ferrimagnetic alloy with $z = 3$ and $p = 0.495$ obtained from the EFT. (*a*) The $|M|$ curves of the systems when two sets of $(J_B/J_A, D/J_A, J_{AB}/J_A)$ are selected: (0.5, −3.0, −3.055) for the full curve and (0.1, −2.0, −2.455) for the broken curve. (*b*) The $|M|$ curves of the systems with two sets of $(J_B/J_A, D/J_A, J_{AB}/J_A)$: (0.5, −3.0, −3.2) for the full curve and (0.1, −2.0, −2.55) for the broken curve.

take forms different from those of the systems with $p = 0.495$ in figures 7 and 8. Some typical results are depicted in figure 9. In figure 9(*a*), the $|M|$ curves exhibit three and two compensation points for the two sets of $(J_B/J_A, D/J_A, J_{AB}/J_A)$, namely (0.1, −0.448, −0.5) for the full curve and (0.1, −0.4502, −0.5) for the broken curve, although each saturation magnetization at $T = 0$ K is $|M| = 0.0$. In figure 9(*b*), on the other hand, the full curve for the system with $(J_B/J_A, D/J_A, J_{AB}/J_A) = (0.1, -0.447, -0.5)$ may show two (weak and sharp) maxima and a weak minimum below the compensation point. The broken curve for (0.1, −0.451, −0.5) shows two maxima and a minimum, and reduces to zero at the Curie point. Comparing figure 9(*a*) with figure 9(*b*), one can see that the temperature dependence of $|M|$ in the system with $p = 0.5$ may dramatically change its features by a small variation of the parameter D/J_A.

Finally, all results obtained in this section, namely figures 6–9, have not been predicted in the Néel theory of ferrimagnetism [1].

6. Conclusions

In this work, we have studied the magnetic properties of a disordered ferrimagnetic binary alloy on the basis of two theoretical frameworks, namely the standard mean-field theory (MFA) and the effective-field theory with correlations (EFT). The EFT corresponds to the Zernike approximation, superior to the standard MFA. It can be easily proved by taking the limit ($D/J_A \to -\infty$) in the formulation, as noted in section 2. In the two formulations, the theoretical results depend only on the coordination number z but not on the dimensionality. However, the disordered ferrimagnetic binary Ising alloy model normally simulates well the topologically disordered ferrimagnets, such as amorphous ferrimagnets, in which many physical quantities depend on the coordination number [3]. Moreover, one should notice that the mean-field results of this work can also be obtained for any z although we took the case of $z = 3$ for comparision with those of the effective-field theory.

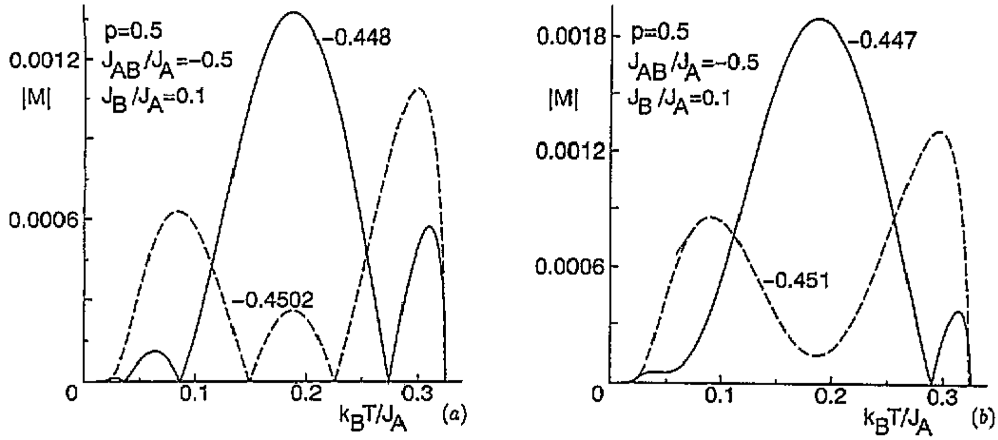


Figure 9. Some characteristic temperature dependences of $|M|$ in the ferrimagnetic alloy with $z = 3$ and $p = 0.5$ obtained from the EFT. (*a*) The $|M|$ curves for systems with two sets of $(J_B/J_A, D/J_A, J_{AB}/J_A)$: (0.1, −0.448, −0.5) for the full curve and (0.1, −0.4502, −0.5) for the broken curve. (*b*) The $|M|$ curves for systems with two sets of $(J_B/J_A, D/J_A, J_{AB}/J_A)$: (0.1, −0.447, −0.5) for the full curve and (0.1, −0.451, −0.5) for the broken curve.

As discussed in section 4, a disordered ferrimagnetic binary alloy with a concentration near $p = 0.75$ may show two compensation points as well as an outstanding feature in the thermal variation of the resultant magnetization, such as the broken curves in figures 4(*b*) and 5(*b*). In section 5, we have also found three compensation points as well as some interesting thermal variations in the total magnetization curves for the system with a concentration near $p = 0.5$. These results have not been predicted in the Néel theory of ferrimagnetism. In particular, we have to change our concept of only one compensation point in ferrimagnetic materials, normally written in standard textbooks of magnetism. We hope that the results obtained in this work may be helpful when the experimental data of ferrimagnetic materials are analysed. From the technological point of view, the existence of many compensation points may be useful for thermomagnetic writing and erasing, because of the high coercivity around the points.

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